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A PROPOSAL OF ROBUST DESIGN METHOD APPLICABLE TO DIVERSE DESIGN PROBLEMS

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ABSTRACT:

The need for robust design that enables the products to ensure robust performance in diverse surroundings has been focused. Robust design guarantees robust performance in manufacturing variation of products and diverse use environment. In this study, a robust design method applicable to diverse design problems was proposed. In the proposed method, to use the robust index R, the weighted robust index R_W, and the adjusted robust index R_A as indexes of robustness, the proposed method is applicable to design problems in which distribution pattern of objective characteristic y is non-normal distribution with multi peak, the control factor x is adjustable. And, to confirm the effectiveness, the proposed method was applied to a public seat design. As a result of the verification, the solution obtained by the proposed method was better than the existing. It was confirmed that the proposed method was superior to the existing methods.

KEYWORDS: ROBUST DESIGN, ROBUST INDEX, SEAT DESIGN

1. INTRODUCTION

The environment surrounding design product has been diverse due to increased individuation of the user needs and globalization of markets. Therefore, the need for robust design that enables the products to ensure robust performance in diverse surroundings has been focused. Robust design guarantees robust performance in manufacturing variation of products and diverse use environment. In robust design, many methods including Taguchi method (Taguchi, G. 1993) have been proposed. However, the existing robust design methods (RDMs) are not applicable to all design problems as most RDMs are based on the premise that objective function, which is a relation expression of factors and objective characteristic, is linear. Therefore, it is considered that there are design problems to which existing RDMs are not applicable.

The objective of this study is to clarify the characteristics of design problems to which existing RDMs are not applicable, to propose a RDM which is applicable to these diverse design problems, and to confirm the possible application and effectiveness of the proposed method by applying a design case.

2. RDMS AND THEIR PROBLEMS

RDMs can be classified into the methods based on experiment and the methods based on simulation.

2. 1. RDMS BASED ON EXPERIMENTS

RDMs based on experiments have their origin in the design of experiment. RDMs evaluate and improve the robustness of function on objective characteristic y, which is a physical value to express the design objective, by using the data obtained by full factorial designs experiment or orthogonal array experiment. Specifically, the first step is to get the average and standard deviation of y's experimental value by experimenting with each level of control factor x, which is a factor that designer is able to control, and noise factor z, which is a factor that designer is unable to control. Using these data, the second step is to make optimization of two aims: to minimize the difference between the objective characteristic value and the target value τ, and to minimize the variations of *y*. (Fig. 1) shows an example of orthogonal array and mathematical formula of concept based on experiment.

In this study, four methods, Taguchi's method, Otto's method (Otto, K.N. and Antosson, E.K. 1993), Sundaresan's method (Sundaresan, S. Ishii, K. and Houser, D.R. 1991), and Yu's method (Yu, J.C. and Ishii, K. 1993) were introduced as RDMs based on experiment.

2. 2. RDMS BASED ON SIMULATIONS

RDMs based on simulation can be further classified into the methods for transforming objective functions and the methods for transforming constraint functions, which expresses the relation between factors and constraint characteristics. Methods for transforming objective functions evaluate and improve the robustness of *y*. The first step is to transform the objective functions into different function based on the fluctuation of fluctuant factors (control factors and noise factors). The second step is to make optimization of two aims: to minimize the difference between the objective characteristic value and the target value, and to minimize the variations of *y* with the variations of fluctuant factors. As a result of these processes, a robust design solution which has the variations of *y* smaller than the existing design solution is obtained (Fig. 2).

In this study, seven methods, Ramakrishnan's method (Ramakrishnan, B. and Rao, S.S. 1996), Belegundu's method (Belegundu, A.D. and Zhang, S. 1992), Arakawa's method (Arakawa, M. and Yamakawa, H. 1995), Wilde's method (Wilde, D.J. 1992), Zhu's method (Zhu, J. and Ting,

Number of factors

Number of experiments	1	2	3	4	5	6	7	8	*y*
1	1	1	1	1	1	1	1	1	y_1
2	1	1	2	2	2	2	2	2	y_2
3	1	1	3	3	3	3	3	3	y_3
4	1	2	1	1	2	2	3	3	y_4
5	1	2	2	2	3	3	1	1	y_5
6	1	2	3	3	1	1	2	2	y_6
7	1	3	1	2	1	3	2	3	y_7
8	1	3	2	3	2	1	3	1	y_8
9	1	3	3	1	3	2	1	2	y_9
10	2	1	1	3	3	2	2	1	y_{10}
11	2	1	2	1	1	3	3	2	y_{11}
12	2	1	3	2	2	1	1	3	y_{12}
13	2	2	1	2	3	1	3	2	y_{13}
14	2	2	2	3	1	2	1	3	y_{14}
15	2	2	3	1	2	3	2	1	y_{15}
16	2	3	1	3	2	3	1	2	y_{16}
17	2	3	2	1	3	1	2	3	y_{17}
18	2	3	3	2	1	2	3	1	y_{18}

Minimize

$$\sum_{i=1}^{n}(y_i-\tau)$$

$$\sum_{i=1}^{n}\left|y_i-\frac{1}{n}\sum_{i=1}^{n}(y_i)\right|$$

y : Objective characteristic (experimental value)
n : Number of experiment
τ : Target value

Figure 1: Concept of RDMs based on experiment.

K.L. 2001), Gunawan's method (Gunawan, S. and Azarm, S. 2004), and Eggert's method (Eggert, R.J. 1991) were introduced as methods for transforming objective function.

Next, methods for transforming constraint function evaluate and improve the robustness of constraint characteristic *c*, which is a constrained physical value, by transforming the constraint function into different function based on the fluctuation of fluctuant factors and configuring the constraint condition into a stringer one. As a result of these processes, a robust design solution which *c* does not depart from its feasible area with fluctuation of fluctuant factors is obtained (Fig. 3).

In this study, six methods, Parkinson's method (1), Parkinson's method (2) (Parkinson, A. 1995), Arakawa's method, Sundaresan's method, Eggert's method and Yu's method were introduced as methodsfor transforming constraint function.

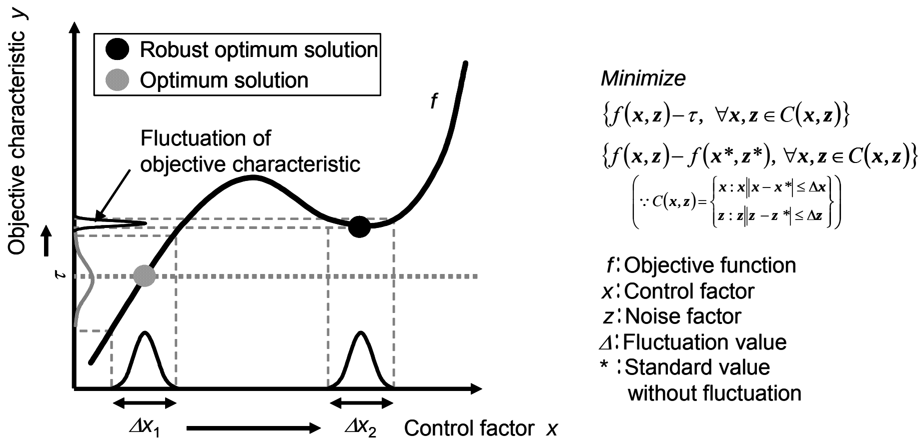

Figure 2: Concept of methods for transforming objective function.

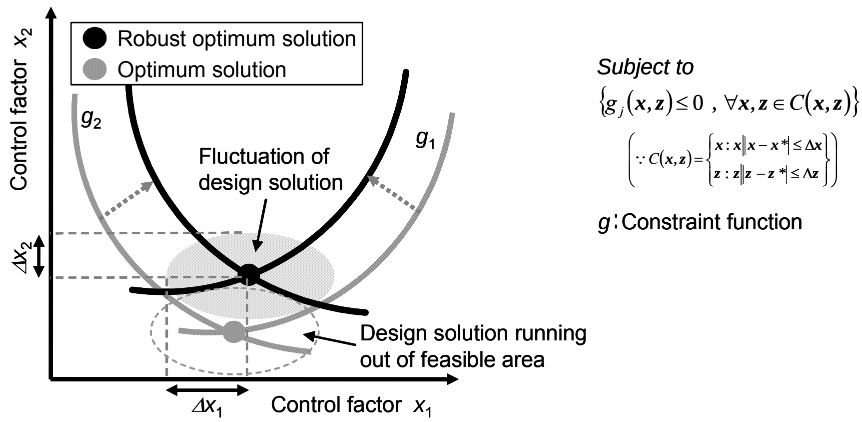

Figure 3: Concept of methods for transforming constraint function.

2. 3. PROBLEMS ASSOCIATED WITH RDMS

For assessment above methods, seven characteristics of design problem were identified: linearity of function, differentiability and monotonicity of function, distribution pattern of fluctuant factors, independence of fluctuant factors, existence of tuning factors, units of fluctuant factors, quality of weight of objective characteristic value, and existence of adjusted factor, which is the control factor having variable range. Then existing methods were assessed if they were applicable to these problems. (Table 1) shows the result of this assessment. B rating indicates that the robust method is applicable to the design problem, and A rating indicates that the method is applicable to the problem effectively with little calculation amount. In addition, C rating indicates that the method is applicable to the problem except when y is nominal-the-better characteristic. The result of assessment confirmed that these methods are not applicable to design problems in which a function is non-linear and fluctuant factors are dependent, weight of objective characteristic value is nonuniform, or control factor is adjustable. For example, in the case of considering probability distribution of *y*, existing methods are applicable to problems in which function is non-liner and the sum of squares of *y* is normal distribution or bilaterally-symmetric uniform as shown in (Fig. 4(a)).

Table 1: Assessment table of problems for RDMs.

RDMs / Characteristics of design problems		RDMs based on experiment				RDMs based on simulation												
						RDMs for transforming objective function							RDMs for transforming constraint function					
		Taguchi's method	Otto's method	Sundaresan's method	Yu's method	Ramakrishnan's method	Belegundu's method	Arakawa's method	Wilde's method	Zhu's method	Gunawan's method	Eggert's method	Parkinson's method (1)	Parkinson's method (2)	Arakawa's method	Sundaresan's method	Eggert's method	Yu's method
Linearity of function	Possible to apply linearization	A	A	A	A	A	A	A	A	A	A	B	A	A	A	A	B	B
	Impossible to apply linearization											B					B	B
Differentiability and monotonicity of function	Differentiable	B	B	B	B	A	A	A	B	A	B	A	A	A	A	B	A	B
	Indifferentiable and monotone increase (decrease)	B	B	B	B				A		B					A		B
	Indifferentiable and not monotone increase (decrease)	B	B	B	B						B							B
Distribution pattern of fluctuant factors	Bilaterally-symmetric uniform	B	B	B	B		A	B	B	A	A			A	B	B		B
	Not bilaterally-symmetric uniform	B	B	B	B			B	B						B	B		B
	Normal		B		B	B						B	B				B	B
Independence of fluctuant factors	Independent	A	B	A	B	A	A	A	A	A	A	A	A	A	A	A	A	B
	Dependent		B		B													B
Tuning factors	Existent	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B
	Nonexistent	C	C	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B
Units of fluctuant factors	Identical unit	B	B	B	B	B	B	B	B	A	A	B	B	B	B	B	B	B
	Nonidentical unit	B	B	B	B	B	B	B	B			B	B	B	B	B	B	B
Weight of objective characteristic	Uniform	B	B	B	B	B	B	B	B	B	B	B						
	Nonuniform																	
Adjusted control factor	Existent												B	B	B	B	B	B
	Nonexistent	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B

On the other hand, it is conceivable that distribution pattern of *y* is often non-normal in actual design problems. In the case where the function is strong non-liner, distribution pattern of *y* is non-normal as shown in (Fig. 4(b)). In addition, in the case where multi functions are fluctuant stochastically, distribution pattern of *y* is non-normal distribution with multi peak as shown in (Fig. 4(c)) because of superposition of multi distribution. In the second case, there are a few errors between distribution pattern and normal distribution; moreover, it is possible to apply properly with Eggert's method to consider chi-square distribution or gamma distribution. However, in the third case, there are big errors between distribution pattern and normal distribution, and it is difficult to relate to other distribution. Moreover, there is no method to proper evaluate the design problem with adjusted mechanism as shown in (Fig. 5).

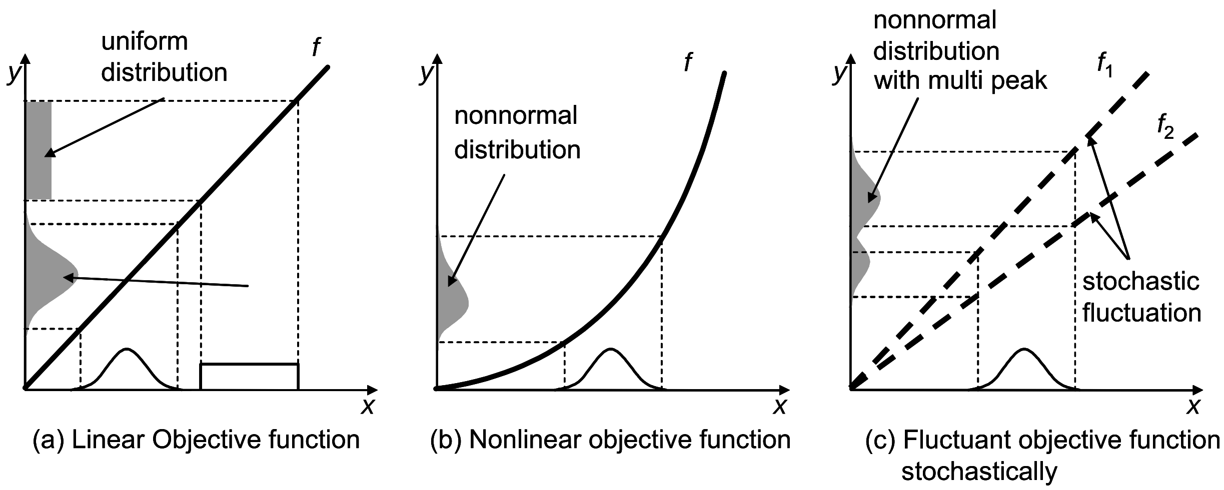

Figure 4: Relation between objective function and probability distribution of *y*.

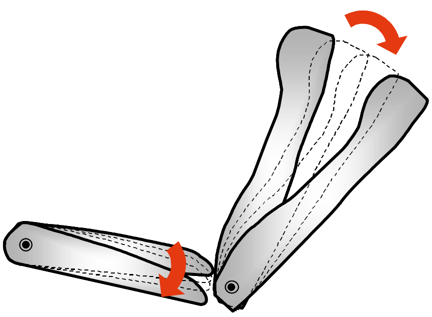

Figure 5: Adjusted mechanism.

3. A PROPOSAL OF ROBUST DESIGN METHOD APPLICABLE TO DIVERSE DESIGN PROBLEMS

The new method applicable to diverse design problems named RDM was proposed, which is applicable to all design problems including the problems to which existing method was not applicable as indicated in the second paragraph. In the proposed method, robustness was defined as the feasibility of the objective characteristic value being within tolerance. By expressing this concept with probability density function of *y*, the proposed method was applicable to above diverse design problems. Here are the evaluation indexes of robustness used in the proposed method.

3. 1. The robust index *R*

The robust index *R* was the feasibility of the objective characteristic value being within tolerance, and was calculated to integrate probability density function of *y*. The feasibility is expressed as follows:

$$R = \int_{y_l}^{y_u} p(y)dy \tag{1}$$

where *p* is probability density, *p*(*y*) is probability density function of the *y*, y_l is the lower tolerance limit, y_u is the upper tolerance limit, and τ is the target value. (Fig. 6) shows the concept of *R*. *R* is the evaluation index to minimize the sum of squares of *y* because it evaluates robustness by using the feasibility of the objective characteristic value being within tolerance. In this study, *R* was calculated using the Monte Carlo method. First, a random number of fluctuant factors were generated based on the probability density function of their fluctuation. Second, objective characteristic value was calculated based on a random number of them. Finally, *R* is calculated as follows:

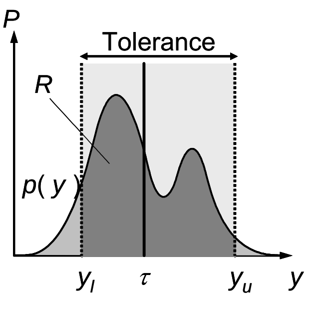

Figure 6: Concept of the robust index *R*.

$$R = \frac{1}{s}\sum_{i=1}^{s} M_i \quad \left(M_i = \begin{cases} 1 & (y_l \le y_i \le y_u) \\ 0 & (otherwise) \end{cases} \right) \tag{2}$$

where *s* is the number of samples generated for a random number of fluctuant factors.

3. 2. The weighted robust index R_W

Here, since *R* is only able to evaluate if the objective characteristic value is within tolerance or not, *R* is not able to evaluate the weight of the objective characteristic value. However, since a design problem has a target value in many cases, the weight of the objective characteristic value increases as the *y* approaches the target value. In this study, the weight of the objective characteristic value was expressed as a weighting function. R_W that enables the weight of the objective characteristic value to be evaluated is expressed as follows:

$$R_W = \int_{y_l}^{y_u} w(y)p(y)dy \tag{3}$$

where *w* is the weight of the objective characteristic value, and *w*(*y*) is the weighting function of the objective characteristic value. (Fig. 7) shows the concept of R_W. R_W is the evaluation measure to optimize two aims, to minimize the sum of squares of *y* and to maximize the weight of the objective characteristic value because it evaluates the feasibility of the objective characteristic value being within tolerance and the weight of the objective characteristic value. In this study, R_W was calculated using the Monte Carlo method as in the case of *R*. R_W is calculated as follows:

$$R_W = \frac{1}{s}\sum_{i=1}^{s} M_i \quad \left(M_i = \begin{cases} w(y) & (y_l \le y_i \le y_u) \\ 0 & (otherwise) \end{cases} \right) \tag{4}$$

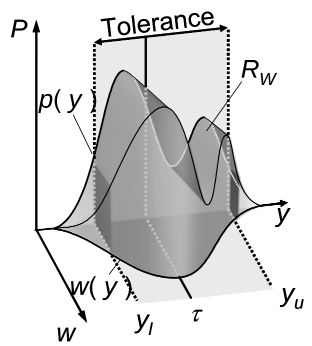

Figure 7: Concept of the weighted robust index R_W.

R and R_W are used depending on the need to consider the weight of objective characteristic value.

3. 3. The adjusted robust index R_A

The adjusted robust index R_A is the evaluation measure to add the concept of adjusted factor to R. Specifically, in the case where control factor has variable range, R_A is the feasibility of the objective characteristic value being within tolerance for each control factor value t_i in the range, and is expressed as the ratio of the sum of sets of combination of fluctuant factors in the case that objective characteristic value being within tolerance in entire set.

$$R_A = P\left[\bigcup_{t_i} \{C(\boldsymbol{x}, \boldsymbol{z}) | y_l \le f(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{t}_i) \le y_u\}\right] \tag{5}$$

where i is level of adjusted factor, $P[A]$ is the feasibility to happen event A. (Fig. 8) shows the concept of R_A. R_A is able to evaluate robustness in case where control factor has variable range. In addition, it can calculate variable range of control factor for additional design with adjusted mechanism to get more robustness, when design solution calculated by using R do not have enough robustness. As some examples of adjusted mechanism, design of automobile seat back structure and seat slide structure follow. R_A was calculated using the Monte Carlo method, as in the case of R and R_W. First, a random number of fluctuant factors were generated. Second, each combination of generated fluctuant factors and objective characteristic value in the case of each combination were calculated. Third, the case any of adjusted factors exists where objective characteristic value being within tolerance was calculated 1 otherwise was 0. Finally, the sum of them was divided by a random number. R_W is calculated as follows:

$$R_A = 1 - \frac{1}{s}\sum_{i=1}^{s} M_i \quad \left(M_i = \begin{cases} 1 & \left(\exists \boldsymbol{t} \in S \middle| |\boldsymbol{t} - \boldsymbol{t}^*| = \Delta\boldsymbol{t};\ y_l \le f(\boldsymbol{x}_i, \boldsymbol{z}_i, \boldsymbol{t}) \le y_u\right) \\ 0 & (otherwise) \end{cases} \right) \tag{6}$$

4. CASE APPLICATIONS

Finally, to confirm the effectiveness, the proposed method was applied to a public seat design preventing hip-sliding force F_{HS} which is a source of discomfort with respect to sitting posture. Here, cushion angle θ_C which is the main factor to control F_{HS} was selected as design object

(Fig. 9). In the process of calculating design solution, to confirm the effectiveness of the proposed method, the solution calculated by Yu's method with average and standard deviation ($\theta_{C(Yu)}$) and the solution calculated by average only without considering sum of squares of y ($\theta_{C(\mu)}$) are compared in addition to the design solution calculated by R and R_W ($\theta_{C(R)}$ and $\theta_{C(RW)}$) (table 2). Moreover, probability distribution of y of each solution was compared (Fig. 10). According to (Table 2), R and R_W in case of $\theta_{C(R)}$ and $\theta_{C(RW)}$ were 5 to 7 percent higher than $\theta_{C(Yu)}$ and $\theta_{C(\mu)}$. This result confirmed that the proposed method advanced the feasibility of the objective characteristic value being within tolerance and the design solutions calculated by the proposed method have higher robustness. This consideration was confirmed by the fact that probability distribution within tolerance of $\theta_{C(R)}$ and $\theta_{C(RW)}$ were more than that of $\theta_{C(Yu)}$ and $\theta_{C(\mu)}$. Moreover, to satisfy more users, proper variable range was calculated using the R_A. As a result of this calculation, the solution satisfied with 95 percent of users with setting the variable range of cushion angle from 16.5 to 19.3; therefore, it was confirmed that the proposed method were superior to the existing methods.

5. CONCLUSIONS

In this study, first, the characteristics of design problems which existing RDMs are not applicable were confirmed. Second, the robust design method applicable to diverse design problems was

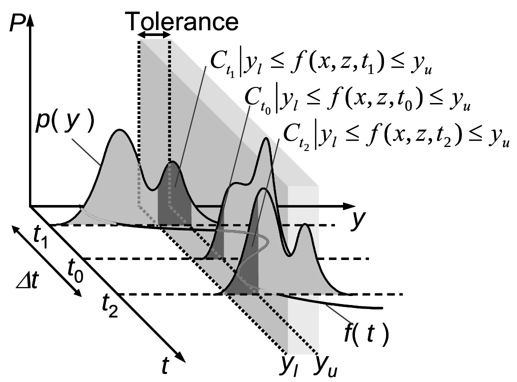

Figure 8: Concept of the adjusted robust index R_A.

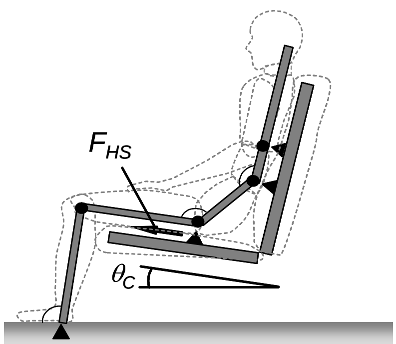

Figure 9: Design object.

proposed as the method applicable to these problems. In the proposed method, the robust index R, the weighted robust index R_W, and the adjusted robust index R_A were used. Moreover, the proposed method was applied to a public seat design, and its possible application and effectiveness were indicated.

Future study is to confirm the general versatility of the proposed method by applying to a wide range of design problems.

6. ACKNOWLEDGMENTS

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Table 2: Design solution of cushion angle.

Optimum solution (deg) \ Measure of robustness	R	RW
$\theta_{C(R)}$=17.9	0.87	3.95
$\theta_{C(RW)}$=17.6	0.85	3.97
$\theta_{C(Yu)}$=18.1	0.80	3.59
$\theta_{C(\mu)}$=18.1	0.80	3.59

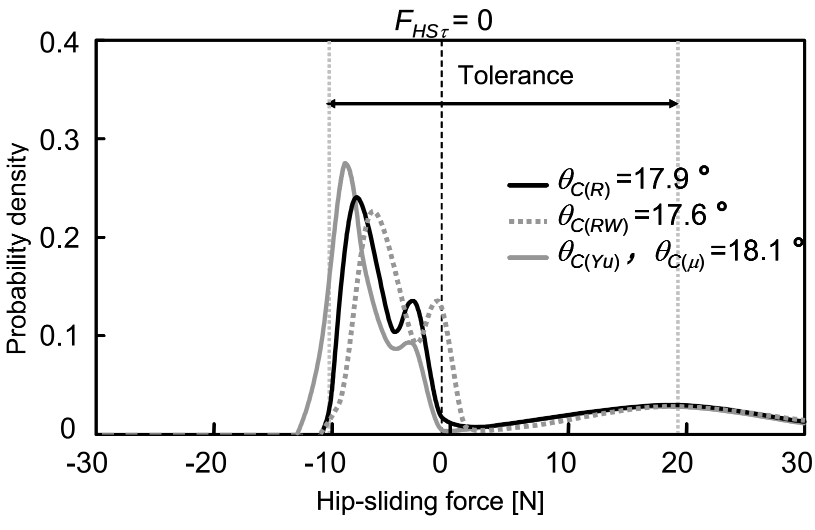

Figure 10: Probability density distribution of y.

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